

Name: _____

Date: _____

Notes

Algebra Section 11.2

Pages 719-726

Goal: “Simplify radicals using the product property”
“Multiply radicals”
“Simplify radicals using the quotient property”
“Rationalize the denominator”
“Add and Subtract Radicals”

Radicals are simplest form when:

1. The number under the radical has no perfect square factors.
2. No variables have an exponent greater than 1.
3. There are no fractions under the radical sign.
4. There are no radicals in the denominator

Properties of Radicals

Product Property: $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$ or $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ so....
 $\sqrt{4 \cdot 9} = \sqrt{4} \cdot \sqrt{9} = 2 \cdot 3 = 6$

Quotient Property: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ or $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ so....

$$\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$$

Examples (Multiplication):

$$\sqrt{9x^2} = \sqrt{9} \cdot \sqrt{x^2} = 3x$$

$$\sqrt{16x^3} = \sqrt{16} \cdot \sqrt{x^2} \cdot \sqrt{x} = 3x\sqrt{x}$$

$$\sqrt{32} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$$

Try These:

$$\sqrt{16x^2}$$

4x

$$\sqrt{4x^4}$$

2x²

$$\sqrt{49x^3}$$

7x√x

$$\sqrt{27}$$

3√3

$$\sqrt{20}$$

2√5

$$\sqrt{64x^2}$$

8x

$$\sqrt{8x^2}$$

2x√2

$$\sqrt{81x^3}$$

9x√x

$$\sqrt{45x^5}$$

3x²√5

$$\sqrt{12x^3y^6}$$

2xy³√3x

Examples (Multiplication):

$$\sqrt{6} \cdot \sqrt{6} = \sqrt{36} = 6$$

$$\sqrt{3x} \cdot 4\sqrt{x} = 4\sqrt{3x \cdot x} = 4 \cdot \sqrt{3} \cdot \sqrt{x^2} = 4x\sqrt{3}$$

$$\sqrt{7xy^2} \cdot 3\sqrt{x} = 3\sqrt{7 \cdot x^2 \cdot y^2} = 3 \cdot \sqrt{7} \cdot \sqrt{x^2} \cdot \sqrt{y^2} = 3xy\sqrt{7}$$

Try These:

$$\begin{array}{l} \sqrt{2} \cdot \sqrt{8} \\ \sqrt{16} \\ 4 \end{array}$$

$$\begin{array}{l} \sqrt{20} \cdot \sqrt{5} \\ \sqrt{100} \\ 10 \end{array}$$

$$\begin{array}{l} \sqrt{5x} \cdot 3\sqrt{x} \\ 3\sqrt{5x^2} \\ 3x\sqrt{5} \end{array}$$

$$\begin{array}{l} \sqrt{8x^2y} \cdot 4\sqrt{2y} \\ 4\sqrt{16x^2y^2} \\ 16xy \end{array}$$

$$\begin{array}{l} 2\sqrt{3a^2b^3} \cdot 5\sqrt{3ab} \\ 10\sqrt{9a^3b^4} \\ 30ab^2\sqrt{a} \end{array}$$

Examples (Division):

$$\sqrt{\frac{13}{100}} = \frac{\sqrt{13}}{\sqrt{100}} = \frac{\sqrt{13}}{10}$$

$$\sqrt{\frac{7}{x^2}} = \frac{\sqrt{7}}{\sqrt{x^2}} = \frac{\sqrt{7}}{x}$$

Try These:

$$\begin{array}{l} \sqrt{\frac{3}{9}} \\ \frac{\sqrt{3}}{\sqrt{9}} \\ \frac{\sqrt{3}}{3} \end{array}$$

$$\begin{array}{l} \sqrt{\frac{5}{n^2}} \\ \frac{\sqrt{5}}{\sqrt{n^2}} \\ \frac{\sqrt{5}}{n} \end{array}$$

$$\begin{array}{l} \sqrt{\frac{a^3}{b^2}} \\ \frac{\sqrt{a^3}}{\sqrt{b^2}} \\ \frac{a\sqrt{a}}{b} \end{array}$$

$$\begin{array}{l} \sqrt{\frac{w^3}{144}} \\ \frac{\sqrt{w^3}}{\sqrt{144}} \\ \frac{w\sqrt{w}}{12} \end{array}$$

$$\begin{array}{l} \sqrt{\frac{16}{4x^4}} \\ \frac{\sqrt{16}}{\sqrt{4x^4}} \\ \frac{4}{2x^2} \end{array}$$

Rationalize the Denominator:

Radicals in the denominator (not perfect square).

Examples:

$$\frac{3}{\sqrt{7}} \quad \text{Multiply by } \frac{\sqrt{7}}{\sqrt{7}} \quad \frac{3}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}}{\sqrt{49}} = \frac{3\sqrt{7}}{7}$$

$$\frac{\sqrt{5}}{\sqrt{2m}} \quad \text{Multiply by } \frac{\sqrt{2m}}{\sqrt{2m}} \quad \frac{\sqrt{5}}{\sqrt{2m}} \cdot \frac{\sqrt{2m}}{\sqrt{2m}} = \frac{\sqrt{10m}}{\sqrt{4m^2}} = \frac{\sqrt{10m}}{2m}$$

Try These:

$$\begin{array}{l} \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \\ \frac{\sqrt{5}}{\sqrt{25}} \\ \frac{\sqrt{5}}{5} \end{array}$$

$$\begin{array}{l} \frac{1}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} \\ \frac{\sqrt{x}}{\sqrt{x^2}} \\ \frac{\sqrt{x}}{x} \end{array}$$

$$\begin{array}{l} \frac{2}{\sqrt{3x}} \cdot \frac{\sqrt{3x}}{\sqrt{3x}} \\ \frac{2\sqrt{3x}}{\sqrt{9x^2}} \\ \frac{2\sqrt{3x}}{3x} \end{array}$$

$$\begin{array}{l} \frac{5}{\sqrt{7n}} \cdot \frac{\sqrt{7n}}{\sqrt{7n}} \\ \frac{5\sqrt{7n}}{\sqrt{49n^2}} \\ \frac{5\sqrt{7n}}{7n} \end{array}$$

$$\begin{array}{l} \frac{\sqrt{2a}}{\sqrt{6a}} \cdot \frac{\sqrt{6a}}{\sqrt{6a}} \\ \frac{\sqrt{12a^2}}{\sqrt{36a^2}} \\ \frac{2a\sqrt{3}}{6a} \end{array}$$

Add and Subtract Radicals:

Radicals are like terms when: when the number under the radical sign (The radicand) is exactly the same. Combine like radical terms by adding or subtracting the coefficient.

Examples:

$$\begin{aligned} 4\sqrt{10} + \sqrt{13} - 9\sqrt{10} \\ 4\sqrt{10} - 9\sqrt{10} + \sqrt{13} \\ -5\sqrt{10} + \sqrt{13} \end{aligned}$$

$$\begin{aligned} 5\sqrt{3} + \sqrt{48} \\ 5\sqrt{3} + \sqrt{16 \cdot 3} \\ 5\sqrt{3} + 4\sqrt{3} \\ 9\sqrt{3} \end{aligned}$$

Try These:

$$\begin{aligned} 7\sqrt{14} + \sqrt{21} - 4\sqrt{14} \\ 3\sqrt{14} + \sqrt{21} \end{aligned}$$

$$\begin{aligned} 2\sqrt{7} + 3\sqrt{63} \\ 2\sqrt{7} + 9\sqrt{7} \\ 11\sqrt{7} \end{aligned}$$

$$\begin{aligned} 2\sqrt{7} + \sqrt{28} \\ 2\sqrt{7} + 2\sqrt{7} \\ 4\sqrt{7} \end{aligned}$$

Distribute: (combine like terms if possible)

F.O.I.L = first, outer, inner, last

Example:

$$\begin{aligned} \sqrt{5}(4 - \sqrt{20}) \\ 4\sqrt{5} - \sqrt{100} \\ 4\sqrt{5} - 10 \end{aligned}$$

$$\begin{aligned} (\sqrt{7} + \sqrt{2})(\sqrt{7} - 3\sqrt{2}) \quad \text{F.O.I.L} \\ (\sqrt{7} \cdot \sqrt{7}) + (\sqrt{7} \cdot -3\sqrt{2}) + (\sqrt{2} \cdot \sqrt{7}) + (\sqrt{2} \cdot 3\sqrt{2}) \\ 7 + -3\sqrt{14} + \sqrt{14} + 3\sqrt{4} \\ 7 - 2\sqrt{14} + 3 \cdot 2 \\ 1 - 2\sqrt{14} \end{aligned}$$

Try These:

$$\begin{aligned} \sqrt{3}(2 + \sqrt{12}) \\ 2\sqrt{3} + \sqrt{36} \\ 2\sqrt{3} + 6 \end{aligned}$$

$$\begin{aligned} (\sqrt{2} + \sqrt{5})(\sqrt{2} - 3\sqrt{5}) \\ (\sqrt{2} \cdot \sqrt{2}) + (\sqrt{2} \cdot -3\sqrt{5}) + (\sqrt{5} \cdot \sqrt{2}) + (\sqrt{5} \cdot -3\sqrt{5}) \\ \sqrt{4} - 3\sqrt{10} + \sqrt{10} - 3\sqrt{25} \\ 2 - 3\sqrt{10} + \sqrt{10} - 15 \\ -2\sqrt{10} - 13 \end{aligned}$$

$$\begin{aligned} \sqrt{2}(3 + \sqrt{2}) \\ 3\sqrt{2} + \sqrt{4} \\ 3\sqrt{2} + 2 \end{aligned}$$

$$(4 - \sqrt{3})(6 + \sqrt{3})$$

$$\begin{aligned} 24 + 4\sqrt{3} - 6\sqrt{3} - \sqrt{9} \\ 24 + 4\sqrt{3} - 6\sqrt{3} - 3 \\ 21 - 2\sqrt{3} \end{aligned}$$

$$\sqrt{6}(7\sqrt{3} + 6)$$

$$\begin{aligned} 7\sqrt{18} + 6\sqrt{6} \\ 21\sqrt{2} + 6\sqrt{6} \end{aligned}$$

$$(3\sqrt{5} + 7)^2$$

$$\begin{aligned} (3\sqrt{5} + 7)(3\sqrt{5} + 7) \\ 9\sqrt{25} + 21\sqrt{5} + 21\sqrt{5} + 49 \\ 45 + 42\sqrt{5} + 49 \\ 94 + 42\sqrt{5} \end{aligned}$$